INSTITUT DES HAUTES ÉTUDES
POUR LE DÉVELOPPEMENT DE LA CULTURE, DE LA SCIENCE ET DE LA TECHNOLOGIE EN BULGARIE
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Concours Général de Physiques "Minko Balkanski"

20 mai 2007

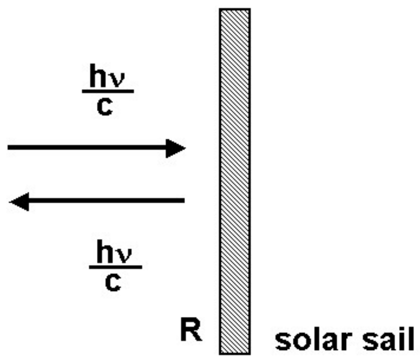
The two exercices are independent.

All the answers must be given in **English** *or in* **French**. *The clarity and the precision will be taken into account for the final note.*

The exam is 4 hours long.

The solar sail

Just like the ships that were once pushed by the wind, in the future we will travel from planet to planet by the help of sails driven by the solar radiation... This is no longer a fiction, but a scenario that is envisioned by a number of scientists and rocket engineers. Indeed, the radiation driven solar sails will not consume any fuel. In this problem we try to understand the way that they function and to estimate somme orders of magnitude.



1. The solar sail is modelled by a mirror of reflectivity R, like in the figure above. A photon flow of energy $h\nu$ is incident on the sail, in the perpendicular direction, where h is the Plank constant and ν the frequency of the radiation.

Let's recall that a photon of energy $h\nu$ carries a momentum p :

$$p = \frac{h\nu}{c} \tag{1}$$

with c the speed of light, $c = 3 \times 10^8\ ms^{-1}$.

1.a) Suppose that the incident photon is *absorbed* by the sail. What is the momentum transferred to the sail ?

1.b) Suppose now that the photon is *reflected* by the sail. What is the momentum transferred to the sail ?

1.c) If the photon is reflected with a probability R and absorbed with a probability $1 - R$, what is the mean momentum transferred by each photon ?

1.d) The photon flow, or the number of photons arriving to the sail per unit surface and unit time is Φ. If the sail surface is S, show that the force F applied to the sail is

$$F = S\Phi\frac{h\nu}{c}(1 + R) \tag{2}$$

2. We are now interested by the launching of the sail. In the following table we plot the solar energy flow, in W/m^2, at the vicinity of a planet, as a function of the planet-sun distance in Astronomical Units A.U. We recall that one astronomical unit is the distance between the Sun and the Earth : 1 A.U. = 149.6×10^6 km.

Planet	Distance to the Sun (A.U.)	Sun energy flow (W/m^2)	Flow $\times$ Distance^2
Mercury	0.387	9134	
Venus	0.723	2617	
Earth	1	1368	
Mars	1.524	589	
Jupiter	5.203	50.53	
Saturn	9.539	15.03	
Uranus	19.182	3.718	
Neptune	30.057	1.514	
Pluto	39.750	0.8657	

2.a) What is the relationship between the energy flow Φ_E and the flow of photons Φ, if the mean energy per photon is $h\nu$? Give then a simplified expression of the radiation force on the sail.

2.b) If the sail is to attain a high speed, is it more advantageous to launch it in the vicinity of Mercury or Neptune ? Why ?

2.c) Fill up the forth column of the table by the product of the energy flow times the square of the planet-sun distance. What is to be noticed ? Provide an explanation.

2.d) Show then that the radiation fore can be written :

$$F = S\frac{A}{cr^2}(1 + R) \tag{3}$$

with r the planet-Sun distance and A a constant that will be calculated numerically, in S.I. units.

3. The force that is obtained in the previous question recalls the gravitational force between the Sun and an object of mass m, that reads :

$$F = -\frac{GMm}{r^2} \tag{4}$$

with G the gravitational constant, $G = 6.67 \times 10^{-11}\ m^3 s^{-2} kg^{-2}$, and M the solar mass, $M \approx 2 \times 10^{30}\ kg$.

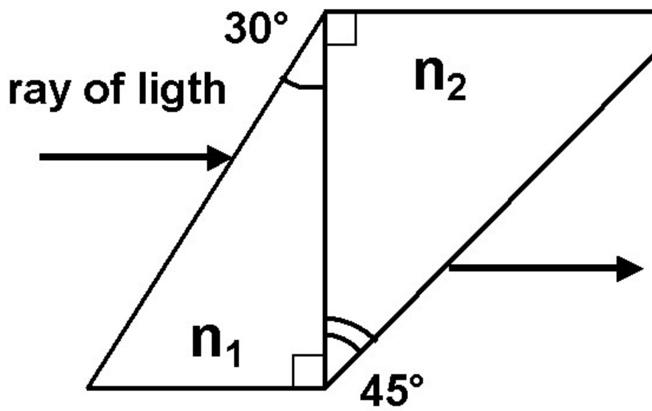
3.a) What are the major differences between the two forces (4) and (3) ?

3.b) A solar sail of mass $m = 100\ kg$ is deployed in the orbit of the Earth. The sail reflectivity is $R = 0.9$. What is the surface of the sail S requested so that the radiation force equilibrates exactly the gravitational force of the Sun ?

3.c) Consider a material of typical density $\rho = 1\ kgm^{-3}$. Does the value of S obtained in the previous question seem realistic to you ?

3.d)* In fact, if the sail is launched from the Earth orbit, it is not necessary to compensate the Sun attraction. Why ? Explain how a solar sail, even with a surface much smaller than the one calculated before, can be successfully employed in inter-planetary trips.

An exercice in optics



Two triangular prisms, with angles 30° and 45° are disposed as in the figure above. A ray of light, that is parallel to the prisms basis, is entering the system, and is leaving in the same direction. What is the refractive index n_2 of the 45° prism if the refractive index of the 30° prism is equal to $n_1 = 2.1$? We recall the trigonometric identity :

$$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \sin(\beta)\cos(\alpha) \qquad (5)$$

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